Turn in first day of School!

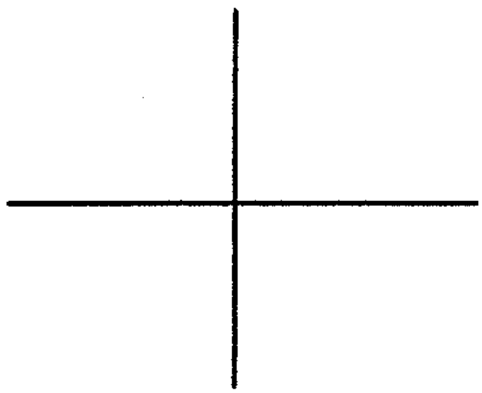
AP Calc BC
Summer work

Limits and Continuity

1. Show three ways that the limit as x approaches 0 of $\dfrac{x}{1-\sqrt{1+x}}$ is -2

Show using a table: Use at least 6 values, 3 on either side of zero

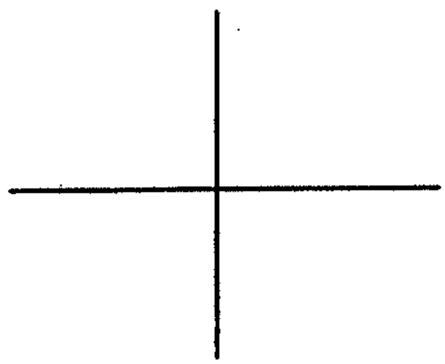
Show using a graph: Write the boundaries of your window.



Show analytically using algebra.

2. Suppose $f(x) = x^2$ if $x>3$, and $5-x$ if $x\leq 3$

a) Graph f(x). Show your window. Include the behavior of f(x) near x=3.



b) Find $\lim_{x\to 3^-} f(x)$

c) Find $\lim_{x\to 3^+} f(x)$

3.

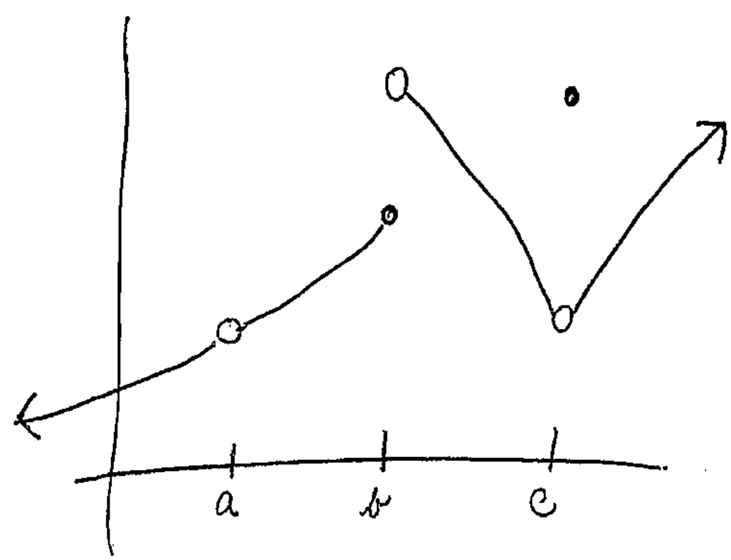
a) What is

$$\lim_{x\to 2} \frac{x-4}{2x+1}$$

b) What is

$$\lim_{x\to \infty} \frac{x-4}{2x+1}$$

4. Suppose f(x) has the following graph:



Tell which of the conditions for Continuity (NOT which TYPE of Discontinuity) is not satisfied at each Of the following values. Use correct Mathematical notation.

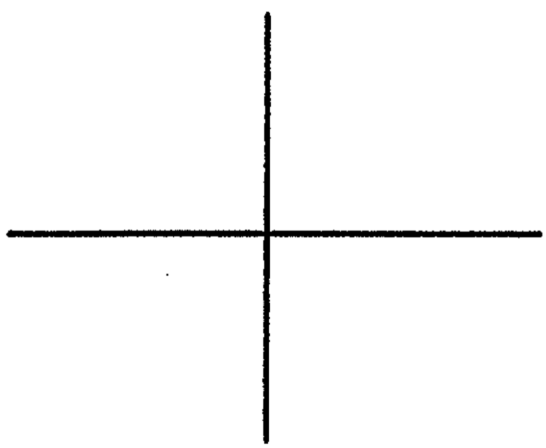
a) x=a

b) x= b

c) x=c

5. Suppose h(x) is continuous on [3,5] and h(3) = 7 and h(5) = 4.
What theorem allows you to conclude that h(x) = 6 for some x in (3,5)?

Draw a function with h(3) = 7 and h(5) = 4 for which there is no such x if h(x) is NOT continuous. on the interval [3, 5]   (means the same as $3 \le x \le 5$



6. Prove using the limit definition of a derivative that if $f(x) = x^2$, then $f'(x) = 2x$

7) Given the following chart f(x) continuous on [2,4] and twice differentiable on (2,4):

X	F(x)	F'(x)	F''(x)
2	-1	3	-3
3	0	0	1
4	3	-2	0
5	5	1	2

True or False: Justify your answer:

a) F(x) must have a maximum value at (3,0).

b) F(x) must have a point of inflection at (4,3)

c) F(x) is an increasing function on the interval [2,5].

Find the equation of the tangent line to F(x) at the point (2,-1).

8) Differentiate: Use your theorems:

$f(x) = 3x^4 - \sqrt{x} + \dfrac{2}{3x^2}$

$g(x) = e^x \sin(x)$

f'(x) =

g'(x)=

$h(x) = \dfrac{\ln(x)}{x}$

$j(x) = e^{(x^2)}$

h'(x) =

j'(x) =

9) Show how to find the minimum value of y=xe^x on (-∞,∞) using y'.
Illustrate your answer with a graph.

10) A particle moves along the x-axis according to $f(t) = t^3 - 6t^2 + 9t$ for $0 \le t \le 6$.

a) When is the particle's velocity equal to zero?

b) What is the smallest value reached on the x-axis for $0 \le t \le 6$? Justify your answer.

c)_Find all intervals on $(0,6)$ for which the acceleration of the particle is positive.

11) Which of the following is true?

a) If f(x) is continuous on [a,b], then f(x) is differentiable on (a,b)

b) If f(x) is differentiable on (a,b), then f(x) is continuous on [a,b].

Give the equation of, or draw a function that disproves that one that you think is not true. Write a sentence with correct mathematical notation describing your claim.

12) A cylindrical tin can must have volume $16\pi\ cm^3$. Find the value of the radius for which the surface area of the can will be a minimum (include both bases of the cylinder in your surface area calculation).

You will also be given a multiple choice activity about pre-calculus — no calculator!