

Turn in first day
of school!

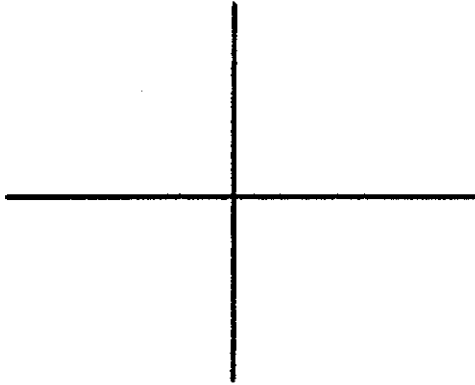
AP Calc BC
Summer work

Limits and Continuity

1. Show three ways that the limit as x approaches 0 of $\frac{x}{1-\sqrt{1+x}}$ is -2

Show using a table: Use at least 6 values, 3 on either side of zero

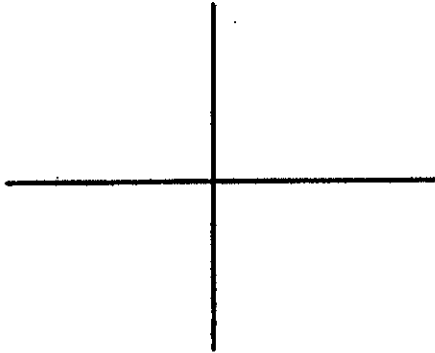
Show using a graph: Write the boundaries of your window.



Show analytically using algebra.

2. Suppose $f(x) = x^2$ if $x > 3$, and $5-x$ if $x \leq 3$

a) Graph $f(x)$. Show your window. Include the behavior of $f(x)$ near $x=3$.



b) Find $\lim_{x \rightarrow 3^-} f(x)$

c) Find $\lim_{x \rightarrow 3^+} f(x)$

3.

a) What is

$$\lim_{x \rightarrow 2} \frac{x-4}{2x+1}$$

b) What is

$$\lim_{x \rightarrow \infty} \frac{x-4}{2x+1}$$

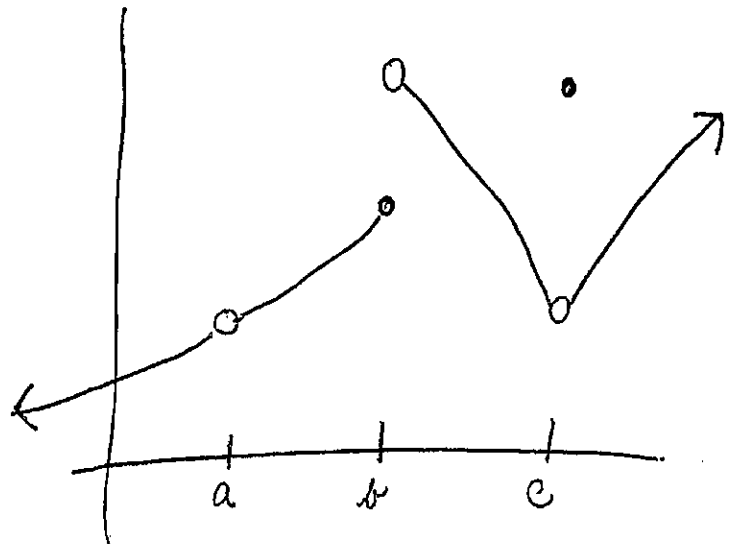
4. Suppose $f(x)$ has the following graph:

Tell which of the conditions for Continuity (NOT which TYPE of Discontinuity) is not satisfied at each of the following values. Use correct Mathematical notation.

a) $x=a$

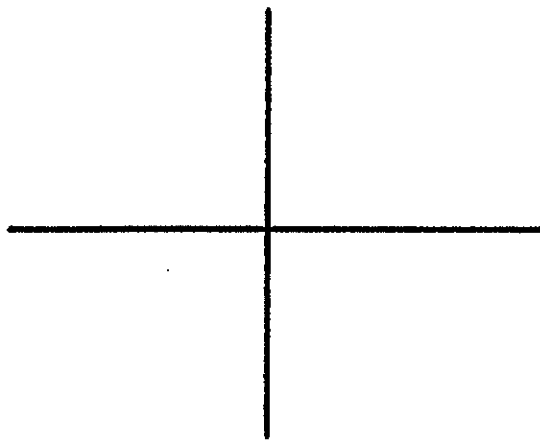
b) $x=b$

c) $x=c$



5. Suppose $h(x)$ is continuous on $[3,5]$ and $h(3) = 7$ and $h(5) = 4$.
What theorem allows you to conclude that $h(x) = 6$ for some x in $(3,5)$?

Draw a function with $h(3) = 7$ and $h(5) = 4$ for which there is no such x if $h(x)$ is NOT
continuous. *on the interval $[3, 5]$ (means the same as $3 \leq x \leq 5$)*



6. Prove using the limit definition of a derivative that if $f(x) = x^2$, then $f'(x) = 2x$

7) Given the following chart $f(x)$ continuous on $[2,4]$ and twice differentiable on $(2,4)$:

X	F(x)	F'(x)	F''(x)
2	-1	3	-3
3	0	0	1
4	3	-2	0
5	5	1	2

True or False: Justify your answer:

- a) $F(x)$ must have a maximum value at $(3,0)$.

- b) $F(x)$ must have a point of inflection at $(4,3)$

- c) $F(x)$ is an increasing function on the interval $[2,5]$.

Find the equation of the tangent line to $F(x)$ at the point $(2,-1)$.

8) Differentiate: Use your theorems:

$$f(x) = 3x^4 - \sqrt{x} + \frac{2}{3x^2}$$

$$g(x) = e^x \sin(x)$$

$$f'(x) =$$

$$g'(x) =$$

$$h(x) = \frac{\ln(x)}{x}$$

$$j(x) = e^{(x^2)}$$

$$h'(x) =$$

$$j'(x) =$$

9) Show how to find the minimum value of $y = xe^x$ on $(-\infty, \infty)$ using y' .

Illustrate your answer with a graph.

10) A particle moves along the x-axis according to $f(t) = t^3 - 6t^2 + 9t$ for $0 \leq t \leq 6$.

a) When is the particle's velocity equal to zero?

b) What is the smallest value reached on the x-axis for $0 \leq t \leq 6$? Justify your answer.

c) Find all intervals on $(0,6)$ for which the acceleration of the particle is positive.

11) Which of the following is true?

- a) If $f(x)$ is continuous on $[a,b]$, then $f(x)$ is differentiable on (a,b)
- b) If $f(x)$ is differentiable on (a,b) , then $f(x)$ is continuous on $[a,b]$.

Give the equation of, or draw a function that disproves that one that you think is not true. Write a sentence with correct mathematical notation describing your claim.

12) A cylindrical tin can must have volume $16\pi \text{ cm}^3$. Find the value of the radius for which the surface area of the can will be a minimum (include both bases of the cylinder in your surface area calculation).

You will also be given a multiple choice activity about pre-calculus — no calculator!

